

Causes of Senior High School Students Errors in Addition, subtraction and Ordering of Fractions

¹Ambrose Kombat, ²Vincent Ninmaal Asigri, ³Joseph Amiyine Adugbire,
⁴Philip Batatia Aketemah

¹St. Bernadette's Technical Institute, Department of Mathematics, Navrongo, Ghana

²St. John Bosco's College of Education, Department of Mathematics and ICT Education, Navrongo, Ghana

³Zamse Senior/High School, Department of Mathematics, Bolgatanga, Ghana

⁴Our Lady of Lourdes Senior High School, Navrongo, Ghana

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Abstract: This study aimed at identifying the errors students commit when ordering, adding and subtracting fractions and the sources of these errors. This qualitative study employed a case study design with a sample size of 78 to help unravel students' errors in fractions. Test and interview were the data collection instruments. Content analysis method was used to identify the errors students commit in fraction test while the interview data was analyzed thematically to ascertain the causes of the errors. It was found from the content analysis that, students committed both conceptual and procedural errors. The interview revealed that, lack of conceptual understanding, overgeneralization on the part of students and the manner in which students were exposed to the BODMAS algorithm were the sources of students' errors. It was therefore, recommended that teachers should always place more emphasis on these areas that students committed the errors to avoid future occurrences and should teach students the applications of mathematics principles and algorithms in a more flexible manner.

Keywords: Errors, Causes of errors, Conceptual, Procedural, Addition, Subtraction, and Ordering.

I. INTRODUCTION

Fractions is one of the most important topics that run across the basic school mathematics curriculum in Ghana (National Council for Curriculum and Assessment & Ministry of Education [NaCCA & MOE], 2019). Understanding the concept of fractions is crucial for the formation and growth of mathematical ideas as well as for aiding in the understanding of other difficult mathematical concepts such as algebra (Booth, Lange, Koedinger & Newton, 2014; Bailey, Siegler & Geary, 2014; Janna & Prahmana, 2019). Also, understanding the notion of fractions aids in problem-solving on daily basis, particularly when percentages, ratios, rates and decimals are involved (Abdul-Halim, Nur-Liyana & Marlina, 2015; Booth, Newton & Twiss-Garrity, 2014; Ndalichako, 2013; Wijaya, 2017; Ayvaz can & Turer, 2018; Van de Walle, Karp & Bay-Williams, 2019).

Despite the importance of fractions in academia and in daily life, students have misconceptions about fractions and experience difficulties in learning it (Van Hoof, Engelen, & Van Dooren, 2021). It is one of the topics or concepts where students frequently err in mathematics (Lemonidis & Pilianidis, 2020; Reinholda et al., 2020; Van Hoof, Engelen, & Van Dooren, 2021; Baidoo, 2019). Studies revealed that, students struggle with fractional addition, subtraction, division and multiplication operations (Baidoo, 2019; Ubah & Banilal, 2018).

For instance, Fitri and Prahmana (2019) investigation into grade seven students' errors in addition of fractions revealed that students committed errors in the addition of fractions and converted mixed fractions into improper fractions wrongly and vice versa. Also, Lestiana, Rejeki and Setyawan, (2017) in their study to identify students' errors in fractions revealed that students committed both conceptual and procedural errors. Furthermore, Morales (2014) in his attempt to analyse students' misconceptions and error patterns in adding and subtracting fractions found that, students could not add and subtract fractions with different denominators correctly. Aksoy and Yazlik (2017) research which also aimed at determining the errors and misunderstanding of students, found that students committed errors in the operations with fractions.

Though some studies have been conducted on the topic in other countries and grade levels using different participants and methodologies, not much is seen in the Ghanaian perspective specifically in the Kasena-Nankana Municipal on the kind of errors students commit in fractions as well as the sources of these errors. Also, much of the studies already carried out on the topic were done at the lower grade levels only hence the need for it to be replicated in the Municipality using Senior High School students who are considered to be in higher grade level.

The following research questions were used to guide the study;

1. what type of errors do students commit in fractions?
2. what are the causes of the error's students commit in fractions?

II. LITERATURE REVIEW

Error Types in Mathematics

According to Brown and Eskow (2016), students' errors can be grouped as follows;

1. Factual or careless error which is committed when students lack or run out of facts. For instance, misidentifying operation signs, the value of digits, writing a wrong number etc.
2. Conceptual error which is committed when learners do not understand the concepts connected to the issue such as the concept of ordering fractions, least common denominator, addition and subtraction of fractions.
3. Procedural or computational error which occurs when students apply mathematical procedures inappropriately. For instance, when a student is able to find the least common denominator but fails to use it correctly in adding or subtracting fractions.

Causes of Errors in Mathematics and Fractions

Delastri and Lolang (2023) study on students conceptual and procedural errors in solving algebraic problems found students errors are linked to misconceptions the students have about certain concepts. Also, Andriani et al., (2021) also found that students committed errors due to mis-generalization. The students generalized what happened in other concepts into the new situation which led them into committing a procedural error. Moreso, Makamure and Jojo (2022) study pre-service teachers committed errors in first order ordinary differential equations as a result of misapplication of natural algorithms. Herholdt and Sapire (2014) on the other hand concluded that lack of conceptual understanding led elementary school students into committing mistakes in early grade mathematics.

Errors Students Commit and the Difficulties They Face in Fractions

Lestiana, Rejeki and Setyawan (2016) study revealed that some students struggled to accurately compare and add when solving problems involving fractions. To them, students used improper strategies, which they classified as procedural and conceptual errors. Lestiana et al., (2016) study and the current study are similar since they both aimed at investigating students' errors in fractions but differs in the sense that the current study was conducted in a different setting using different participants.

Aksoy and Yazlik (2017) research which also aimed at determining the errors and misunderstanding of students found that students committed errors in the operations with fractions. The current study is similar to that of Aksoy and Yazlik (2017) in the sense that both studies investigated errors in fractions and in both cases, content analysis method was used to analysed data but differs from each other when it comes to the participants and the grade level at which they were conducted.

Furthermore, Rosli et al., (2020) study revealed that teacher trainees have poor conceptual understanding of whole-unit, part-whole, arithmetic operations and ordering of fractional values. It was also revealed that, using concrete models, problem solving activities and problem posing activities when teaching had a positive impact on trainees understanding of the subject matter as well as their perspectives and actions or behaviour towards fractions.

In addition, Aliustaoglu, Biber and Tuna (2018) investigations of grade six pupils' misconceptions in fractions revealed that, pupils have erroneous ideas about the part-whole relationship, how fractions are represented on a number line, how to compare fractions and how to perform fractional operations.

To add to that, Alghazo and Alghazo (2017) argued that majority of Saudi Arabian college students hold common misconceptions about fractions and calculations involving fractions, such as thinking that all fractions are always part of one and never greater than one and using cross multiplication to solve multiplication problems involving fractions. It was also found that, only 43% of the Saudi Arabian college students calculated $\frac{3}{5} \div \frac{2}{3}$ correctly. Majority of the students cross-multiplied. Also, only 33% of them were able to pictorially represent $\frac{3}{5} \div \frac{2}{3} = \frac{9}{10}$ correctly.

Again, Morales (2014) in his study too discovered that pupils could not correctly add and subtract fractions. Due to their inability to identify the fractions least common denominator, students were unable to add or subtract fractions with various denominators.

Alkhateeb (2019) study on grade five students' errors and the reasoning associated with the errors revealed that students dealt with fractions as integers and misinterpreted the relation between numerators and denominators with the actual value of the fraction. The current study differs from Alkhateeb (2019) in terms of the participants grade levels, location and the fact that Alkhateeb equally delve into the thinking strategies accompanying the errors.

Abdul-Ghani and Mistima (2018) in their study found that, the mistakes often committed by students when adding fractions is linked to their misconceptions in fractions.

A critical examination of literature revealed that fractional misconceptions exist among students at all grades of learning leading them into committing errors (Alacaci, 2012; Bieber, Tuna & Aktas, 2013; Isik & Kar, 2012; Ojose, 2015; Okur & Cakmak-Gurel, 2016; Onal & Yorulmaz, 2017). A good number of these misunderstanding resulted from the application or transfer of prior knowledge to fractions. Brown and Brown (2013) noted that, generalizations made during whole number education have been transferred incorrectly to fractions.

III. METHODOLOGY

Qualitative research method with case study design was employed in this study to help explore the kind of errors students commit in fractions and the source of the errors. For better generalization of the results, the researchers used two first-year Senior High School students from two intact classes with a student population of 78. These 78 students from the two intact classes were conveniently chosen. Self-developed Fractions Achievement Test (FAT) and interview were used to collect data for this study. The test comprised of four open ended questions out of which two were on ordering of fractions and the other two on addition and subtraction of fractions. The research instruments were given to two experienced mathematics education researchers for scrutiny to ensure validity. The inter-rater reliability approach was used to determine the reliability of the test. The scripts were photocopied and given out to two different ratters for them to rate after which the rated scripts from both ratters were then compared to check consistency in the ratings. The comparison was done script by script and item by item. From the comparisons, it was observed that, there was consistency with just some few disparities in the rated scripts. Also, the scripts from the two ratters were correlated and a correlation coefficient of 0.75 was obtained indicating that the instrument was reliable. The qualitative component of the data was made trustworthy by adhering to Lincoln and Guba (1985) strict criteria (Credibility, Dependability, Confirmability and Transferability). The credibility of the data was established through a process called "member- checking," in which students were given access to the transcribed data and asked to verify its accuracy and consistency with their own experience. The services of a qualitative researcher was employed in the data analysis process and this helped to guarantee dependability. The data was double-checked at every stage of the collection and processing to ensure confirmability and it was made easy to be transferred by giving a thorough account of the study's setting, participants and data gathering process.

Content analysis method was used to analyse the errors students committed when solving the fraction achievement test. Scripts of students were coded as C1, C2, C3, ..., C52 for the class ‘C’ and E1, E2, E3, ..., E50 for the class ‘E’. Students’ responses to the open-ended questions were categories into correct, incorrect and unanswered (Aksoy & Yazlik, 2017). The incorrect category of each question was analysed into details to unravel the errors committed by students in their attempt to answer the questions and the interview data was analysed thematically.

IV. RESULTS AND DISCUSSIONS

Q1. What type of errors do Senior High School students commit in fractions?

This research question was answered by analyzing students’ incorrect answers on the fraction achievement test.

Errors Related to Ordering Fractions with Different Numerators/Denominators

Two open-ended questions were crafted for exploring students’ errors in ordering fractions with different numerators/denominators. Students’ responses are presented under the three categories as correct, incorrect and unanswered in table I.

Table I: Frequencies and Percentages of Responses in Ordering Fractions with Different Numerators/Denominators

Class/Question	Correct		Incorrect		Unanswered	
	f	%	f	%	f	%
C/1	7	17.95	30	76.92	2	5.13
C/2	5	12.82	30	76.92	4	10.26
E/1	6	15.38	32	82.05	1	2.57
E/2	4	10.26	31	79.48	4	10.26

Source: Field data, 2022

From table I, 30 students representing 76.92% of the control class and 32 students representing 82.05% of the experimental class answered question one incorrectly. A critical examination of the incorrect responses revealed that two types of errors were committed with type A error being ordering in order of increasing denominators and type B error being ordering in order of increasing numerators. The frequencies and percentages of incorrect responses in each class belonging to the two types of errors are illustrated in table II.

Table II: Frequencies and Percentages of Respondents Who Committed Type A and B Errors

Class/Question	Type A Error		Type B Error	
	f	%	f	%
C/1	24	80.00	6	20.00
E/1	27	84.37	5	15.63

Source: Field data, 2022

From table II, 24 students representing 80.00% of the incorrect responses from the control class and 27 students representing 84.37% of the incorrect responses from the experimental class committed type A error. That is students ordering fractions in order of increasing denominators when they were actually asked to order it in order of ascendancy. These students lack the concept of size as far as fractions are concern and therefore think that, the value of fractions increases with increasing denominators. Students could have ordered the fractions correctly by either finding the least common denominator or by expressing the fractions into percentages but as it was observed from the way they ordered them, it was clear that these students lack the concept of least common denominator and the concept of expressing fractions into percentages. This led them into committing conceptual error. This finding is similar to Aksoy and Yazlik (2017) when they found that students ordered fractions in other of increasing denominators. The only difference between this finding and that of Aksoy and Yazlik (2017) is the fact that in their case (that particular question) the fractions had the same numerator. Below represents the way such students arranged the fractions in order of ascendancy. $\frac{1}{3}, \frac{2}{5}, \frac{7}{8}, \frac{1}{10}$. For scanned samples of students’ responses with this error, see figure 1.

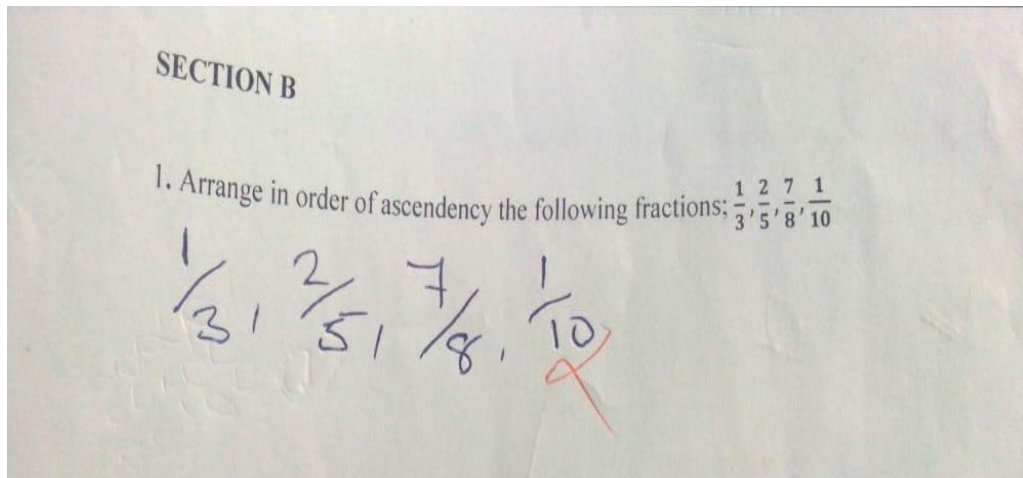


Figure 1: Ordering Fractions in other of Increasing Denominators (Type A Error)

Also, 6 students representing 20.00% of the incorrect responses from the control class and 5 students representing 15.63% of the incorrect responses from the experimental class committed type B error by ordering fractions in order of increasing numerators. These students equally exhibited lack of the concept of size as far as fractions are concern, lack of the concept of least common denominator and expressing fractions as percentages. These students think that, the value of fractions increases with increasing numerators. For the two fractions having the same numerator, students considered the fraction with larger denominator as the largest between the two. This finding is also line with Aksoy and Yazlik (2017). Below was the arrangement of the fractions in ascending order by such students. $\frac{1}{3}, \frac{1}{10}, \frac{2}{5}, \frac{7}{8}$. For scanned scripts of students' responses in this category, see figure 2.

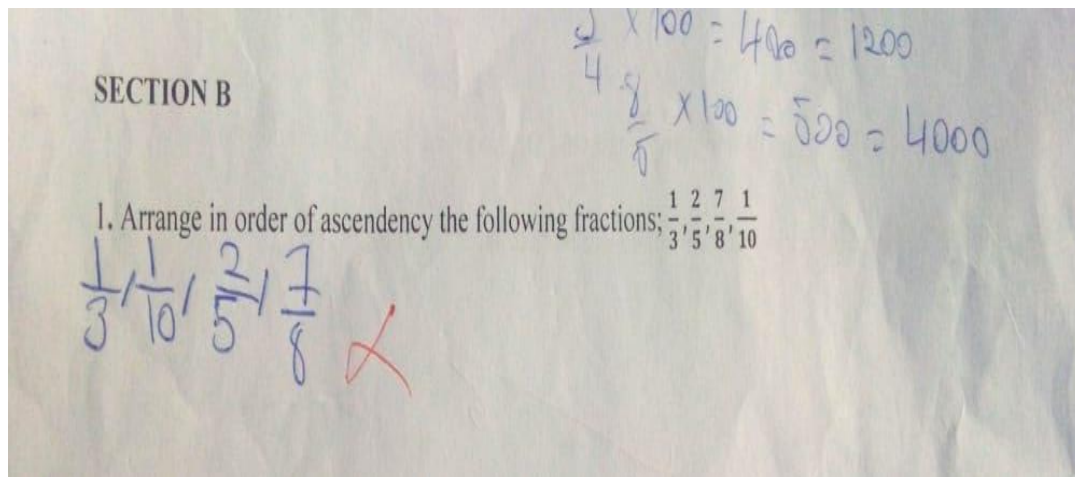


Figure 2: Ordering Fractions in Order of Increasing Numerators (Type B Error)

From table II, it was also found that, 30 students representing 76.92% of the control class and 31 students representing 79.48% of the experimental answered question two incorrectly. Students' responses showed that one type of error was committed. Students ordered fractions in order of either decreasing numerator or decreasing denominator or both. These students think that the value of fractions decreases if either the numerator or the denominator or both numerator and denominator decrease. Here, students again exhibited lack of the concept of size, least common denominator and expressing fractions as percentages. From the ordering of fractions, it was clear that students lack the concept of size as far as fractions is concern thereby committing conceptual error. This finding supports the finding of Aksoy and Yazlik (2017) when they found that 67% of the students considered fraction with large numerator and denominator as fraction having the largest value. These students arranged the fractions in descending order as follows; $\frac{17}{20}, \frac{8}{5}, \frac{3}{4}, \frac{1}{2}$. Samples of students' response to this item are shown in figure 3.

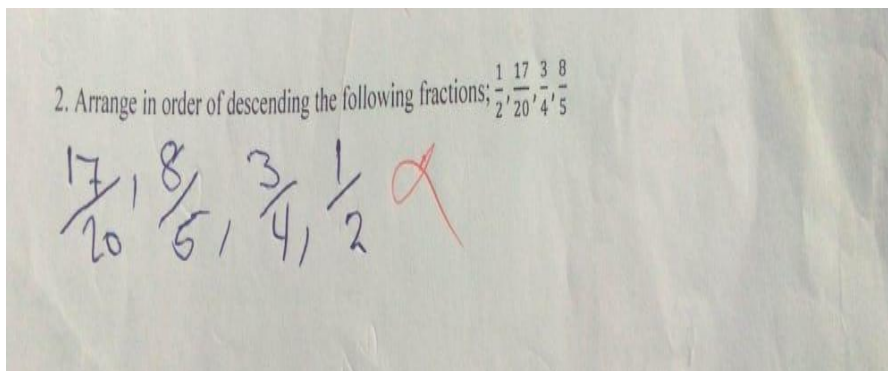


Figure 3: Error in Ordering Fractions in Descending Order

Students Errors Related to Fractions with Both Addition and Subtraction Signs.

Two open-ended questions were developed to explore students’ errors regarding fraction operations involving both addition and subtraction. The frequencies and percentages of students’ responses are presented in table III.

Table III: Frequencies and Percentages of Students Responses Under the Three Categories

Class/Question	Correct		Incorrect		Unanswered	
	f	%	f	%	f	%
C/3	11	28.21	25	64.10	3	7.69
C/4	12	30.77	21	53.85	6	15.38
E/3	9	23.08	28	71.79	2	5.13
E/4	9	23.08	24	61.54	6	15.38

Source: Field data, 2022

In table III, only 11 students representing 28.21% of the control and nine students representing 23.08% of the experimental class answered question three correctly whilst 25 students representing 64.10% of the control class and 28 students representing 71.79% of the experimental class answered it incorrectly. A review of the students’ incorrect response revealed three types of errors with type C being error committed by treating the numerators and denominators as separate entities, type D error being errors committed by the misapplication of BODMAS and type E being errors committed by students who found the least common denominator and then treated the numerators separately from the common denominator. Table IV presents the detailed analysis of the incorrect responses under the three types of errors identified.

Table IV: Frequencies and Percentages of the Incorrect Responses Based on The Three Errors

Class/Question	Type C Error		Type D Error		Type E Error	
	f	%	f	%	f	%
C/3	14	56.00	7	28.00	4	16.00
E/3	19	67.86	6	21.43	3	10.71

Source: Field data, 2022

From table IV, 14 students representing 56.00% of the incorrect responses in the control class and 19 students representing 67.86% of the incorrect responses in the experimental class were type C error. Type C error in this study is the error committed by treating the numerators and denominators as two separate entities or numbers. These students treated the numerators and denominators as independent numbers or entities by subtracting and adding the numerators separately and doing same for the denominators. Students in this category did not know that they were supposed to find the least common denominator of the fractions involved and then using it in the addition or subtraction of the fractions. These students committed a conceptual error. This finding confirms the finding of Alkhateeb (2019) where students responded to $\frac{1}{5} + \frac{2}{5}$ as $\frac{1+2}{5+5} = \frac{3}{10}$. When the students were asked of the meaning of the fraction sign, about 54% of the respondents in his study responded that, “the fraction sign is simply a hyphen or short line (-) that comes between and separate the two numbers”.

Students in this current study probably also saw the fraction sign as a hyphen or short line placed in between the two numbers. It also agrees with that of Lestiana et. al, (2016) findings where students also treated the numerators as separate entities from the denominators making it difficult for them to add/subtract fractions with different denominators correctly. Below is the way the students in the current study responded to the item; $\frac{7}{15} - \frac{1}{3} + \frac{2}{5} = \frac{7-1+2}{15-3+5} = \frac{8}{17}$. Sample of responses are provided in figure 4.

3. Solve $\frac{7}{15} - \frac{1}{3} + \frac{2}{5} = \frac{7}{15} - \frac{1}{3} + \frac{2}{5} = \frac{8}{17}$

Figure 4: Error committed During the Addition/Subtraction of Fractions with Different Denominators

Also, seven students representing 28.00% of the incorrect responses in the control class and six students representing 21.43% of the incorrect responses in the experimental class were type D errors. These students misapplied the principle of BODMAS. These students are of the thinking that, addition must always be performed first before subtraction and for that matter added the second and third fractions first and then subtracted the results from the first fraction. These students equally committed the type C error by doing the top-top and bottom-bottom addition and subtraction. These students are aware of the principle of BODMAS but applied it wrongly which led them into committing both procedural and conceptual errors. This procedural error might have occurred because students did not know that the idea BODMAS can be handled in a flexible manner. Most students were thinking that no matter the nature of the question, addition must always be performed first before subtraction and that division must always be performed first before multiplication and that thinking led students into committing this error. Students at all levels should be made to understand that in BODMAS, when it comes to addition and subtraction or multiplication and division, the idea is whichever operation comes first as we move from left to right of the question should be performed first. Students should also be made to understand that when it comes to addition and subtraction only as found in this item, they could have just found the least common denominator of all the fractions involved in order to avoid this procedural error. This finding also agrees with Alkhateeb, (2019) and Morales (2014). In Alkhateeb (2019) study, 33.55% of the errors committed resulted from the application of algorithms just as the 28.00% and 21.43% of the control and experimental classes who committed errors by wrongfully applying the BODMAS algorithms. Below represents the way the students in this study responded to the item; $\frac{7}{15} - \frac{1}{3} + \frac{2}{5} = \frac{7}{15} - \left(\frac{1}{3} + \frac{2}{5}\right) = \frac{7}{15} - \frac{3}{5} = \frac{4}{7}$. For scanned copies of students' response, refer to figure 5.

3. Solve $\frac{7}{15} - \frac{1}{3} + \frac{2}{5} = \frac{4}{7}$

$\frac{7}{15} - \frac{1}{3} + \frac{2}{5} = \frac{3}{5}$

$\frac{7}{15} - \frac{3}{5} = \frac{4}{7}$

Figure 5: Error Committed by Misapplying BODMAS

Also, four students (16.00%) and three students (10.71%) of the incorrect response from the control and experimental classes respectively were type E errors. Student found the least common denominator but could not use it properly. For instance, after the student found the least common denominator, he or she then subtracted and added the numerators. It can also be argued here that these students dealt with the numerators separately and maintained the biggest denominator. These students committed procedural error since they were able to find the least common denominator but could not use it to solve the question correctly. This finding contradicts one of Morales (2014) findings. Morales (2014) found that students added the numerators and simply wrote the least denominator as $\frac{1}{5} + \frac{3}{8} = \frac{4}{5}$ which is the direct opposite of what is found in this study where students added/subtracted the numerators as in Morales's work but rather wrote the largest denominator as the denominator of the resulting fraction.

Below is the way those students responded to the item; $\frac{7}{15} - \frac{1}{3} + \frac{2}{5} = \frac{7-1+2}{15} = \frac{8}{15}$. Students' responses are shown in figure 6.

Figure 6: Error Committed by Incorrect use of Least Common Denominator

Q2. What are the causes of students' errors in fractions?

This question was answered by analyzing the interview data into themes. For students errors on fractions ordering, two themes emerged. These themes are lack of conceptual understanding on the part of students and overgeneralization of concepts. Students' responses showed that they lacked the concept of expressing fractions into percentages and could not also find the least common denominator of the fractions which could have helped them to do the ordering properly. This finding agrees with Herholdt and Sapire (2014) where it was established that lack of conceptual understanding led elementary school students into committing errors. Below are some excerpts from students buttressing the emerging theme "lack of conceptual understanding".

E13. "I didn't know how to arrange them in ascending order but when I realized that the down numbers were already arranged from smallest to biggest, I just maintained it in that order. I can arrange numbers in ascending order but not when some numbers are down and some being up".

C25. "I arranged as $\frac{17}{20}, \frac{8}{5}, \frac{3}{4}, \frac{1}{2}$ because in that way both the top numbers and down numbers are decreasing since the question said arrange in descending order".

On the part of overgeneralization which also emerged as a factor responsible for the errors students committed, it was realized that students generalized what works in like fractions into unlike fractions. They ordered the fractions by merely concentrating on the values of the numerators as if they were dealing with like fractions. These students did not know that the idea of fractions with bigger numerators being the bigger fractions is only applicable to like fractions. This finding corroborates with the findings of Andriani et al., (2021). In Andriani et al., (2021) study, it was found that students errors were linked to mis-generalizations. Below are some excerpts from the interview with regards to the overgeneralization.

E37: "I arranged in descending order as $\frac{17}{20}, \frac{8}{5}, \frac{3}{4}, \frac{1}{2}$ because $1 < 3 < 8 < 17$. Bigger numerator means bigger fraction".

C17: "I arranged ascending order as $\frac{1}{3}, \frac{1}{10}, \frac{2}{5}, \frac{7}{8}$ because fractions with bigger numerators are usually bigger fractions".

For the addition and subtraction of fractions, three themes emerged from the interview data as the causes of students' errors in fractions. These themes are lack of conceptual understanding, overgeneralization and misapplication of BODMAS algorithm which resulted from how students were exposed to the algorithms at the Primary and Junior High School. On the part of the lack of conceptual understanding, some students responded that they simply added or subtracted the numerators separately from the denominators because they did not know what to do whilst some said they knew they were supposed to find the least common denominator but did not know how to find it. These students lacked the concept of least common denominator as well as the concept of fraction addition and subtraction. This finding is also in agreement with Herholdt and Sapire (2014). Below are excerpts to that effect;

C15. *"I saw some numbers sitting on others. I did not know what to do so I just added and subtracted the up numbers separately and did the same to the down numbers".*

E6. *"I am aware I was supposed to find the least common denominator before working but I did not know how to find it".*

The second factor which emerged from the interview data as a cause of students' errors in addition and subtraction of fractions just as in the ordering of fractions is overgeneralization by students. Students in this case generalized what works for multiplication of fractions into addition and subtraction of fractions which led them into committing procedural errors. This again is in line with the findings of Andriani et al., (2021). Below are interview excerpts backing the theme of overgeneralization.

C39. *"In multiplication we always multiply the top numbers separately and also multiply the down numbers separately and so is addition and subtraction".*

E26. *"Just as in multiplication and division, we are supposed to add or subtract the numerators separately from denominators".*

The last theme which emerged from the interview was the misapplication of the BODMAS algorithm which resulted from how they were taught. Students in this instance exhibited awareness of the algorithm but misapplied it. These students were taught in a way that does not allow flexibility in the use of the algorithm. They did not know that multiplication can actually be dealt with first before division likewise subtraction before addition. This finding concurs with that of Makamure and Jojo (2022) study which found that pre-service teachers committed errors as a result of misapplication of natural algorithms. Excerpts of the interview are shown below;

C12. *"I used BODMAS in solving the question and you know addition is always done first before subtraction".*

E18. *"We were taught that when using BODMAS, addition should always be done first before subtraction".*

V. FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

Findings

Two key findings were revealed at the end of this investigation. These findings include:

1. Learners could not order fractions correctly and also could not add or subtract fractions correctly. When they were made to order fractions in order of increasing size, some of the students ordered the fractions in order of increasing denominators whilst others ordered the fractions in order of increasing numerators. When they were made to add or subtract fractions, most of the learners added or subtracted the numerators separately and repeated same for the denominators. Both conceptual and procedural errors were committed by students.
2. Lack of conceptual understanding, overgeneralization and the way teachers' exposed students to the concept of BODMAS were contributory factors to students' errors in fractions.

Conclusions

The research's findings, led to the following conclusions;

1. Students committed procedural and conceptual errors in the process of ordering, adding and subtracting fractions.
2. The causes of students' errors in ordering, addition and subtraction of fractions are lack of conceptual understanding, overgeneralization and how students were introduced to the concept BODMAS at the basic school.

Recommendation

1. Teachers should plan their lessons in a way that promote conceptual understanding of fractions so as to avoid future occurrences of these and other errors in their students.
2. Teachers should try as much as possible to make their students understand that there is difference between multiplication of fractions and that of addition and/or subtraction of fractions. Also, teachers should ensure that BODMAS algorithm is not taught as if it is a rigid kind of thing. For instance, statements like addition should always be performed first before subtraction and division before multiplication should be avoided. Again, teachers should always emphasize on the differences between like and unlike fractions.

REFERENCES

- [1] Abdul-Ghani, S. N., & Mistima, S. M. (2018). Misconceptions of fraction among middle grade year four pupils at primary school. *Research on Education and Psychology*, 2(1), 111-125.
- [2] Abdul-Halim, A, Nur-Liyana Z. A., & Marlina, A. (2015). Analysis of students' errors in solving higher order thinking skills (hots) problems for the topic of fraction. *Asian Social Science*, 11(21), 133-142.
- [3] Aksoy, N. C., & Yazlik, D. O. (2017). Students' errors in fractions and possible causes of these errors. *Journal of Education and Training Studies*, 5(11), 219.
- [4] Alacaci, C. (2014, pp 63-95). *Misconceptions of students about fractions*. Ankara: Pegem A Publishing.
- [5] Alghazo, Y. M., & Alghazo, R. (2017). Exploring common misconceptions and errors about fractions among college students in Saudia Arabia. *International Education Studies*, 10(4), 133-140.
- [6] Aliustaoglu, F., Tuna, A., & Biber, A. C., (2018) Misconceptions of sixth grade secondary school students on fractions. *International electronic journal of elementary education*, 10(5),591-599.
- [7] Alkhateeb, M. A. (2019). Common Errors in Fractions and the Thinking Strategies That Accompany Them. *International Journal of Instruction*, 12(2), 399-416.
- [8] Andriani, S. P., Triyanto, & Nurhasanah, F. (2021). Procedural error of XIIth grade high school students in solving algebra problems based on Elbrink's theory. IOP Conference Series: *Earth and Environmental Science*, 1796(1).
- [9] Ayvaz Can, A. Turer. (2018). *İlkokul 4. sınıf öğrencilerinin problem çözme başarılarına çözülmüş örnekler yönteminin etkisi* [The effect of worked examples method on problem solving achievement of fourth grade primary school students], (Unpublished doctoral dissertation). Marmara University, Turkey.
- [10] Baidoo, J. (2019). Dealing with grade 10 learners' misconceptions and errors when simplifying algebraic fractions. *Journal of Emerging Trends in Educational Research and Policy Studies (JETERAPS)*, 10(1), 47-55.
- [11] Bailey, D. H., Siegler, R. S., & Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental Science*, 17(5), 775-785.
- [12] Biber, A. Ç., Tuna, A., & Aktaş, O. (2013). The misconceptions of the students about the fractions and the effects of these errors on the solution of fraction problems. *Trakya University Journal of Education*, 3(2), 152-162.
- [13] Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2014). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction*, 25, 24-34.
- [14] Booth, J. L., Newton, K. J. & Twiss-Garrity, L. K. (2014). The impact of fraction magnitude knowledge on algebra performance and learning. *Journal of Experimental Child Psychology*, 118, 110-118
- [15] Brown, E. S., & Brown, E. (2013) Fractions: a concept study. https://www.researchgate.net/publication/305180570_Fractions_A_Concept_Study, Retrived 25th December, 2019
- [16] Brown, J., & Eskow, K. (2016). Mathematics: Identifying and Addressing Students Errors. *The Iris Center*, 3. Retrieved from https://iris.peabody.vanderbilt.edu/case_studies/ics_matherr.pdf.
- [17] Delastri, L., & Lolang, E. (2023). Students' conceptual error and procedural error in solving algebraic problems. *Multicultural Education*, 9(1) 18-24.

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 Vol. 10, Issue 4, pp: (47-57), Month: July - August 2023, Available at: www.noveltyjournals.com

- [18] Fitri, N. L., & Prahmana, R.C.I. (2019). Misconception in fraction for seventh-grade students. *Journal of Physics: Conference series*, 1188(1),012-031.
- [19] Herholdt, R., & Sapire, I. (2014). An error analysis in the early grades mathematics-A learning opportunity?. *South African Journal of Childhood Education*, 4(1) 43-60.
- [20] Işık, C., & Kar, T. (2012). Analyzing problems posed by 7th-grade students for addition operation with fractions. *Elementary Education Online*. 11(4), 1021-1035.
- [21] Jannah, A. F., & Prahmana, R. C. I. (2019). Learning Fraction using the Context of Pipettes for Seventh-Grade Deaf-Mute Student. *Journal for the Education of Gifted Young Scientists*, 7(2), 299–321.
- [22] Lemonidis, C., & Pilianidis, N. (2020). The 8th grade students' competencies in alternating different symbolic representations of rational numbers. *International Electronic Journal of Mathematics Education*, 15(3).
- [23] Lestiana, H. T., Rejeki, S., Setywan, F. (2017). Identifying students' errors in fractions. *Journal of Research and Advances in Mathematics Education*, 1(2), 131-139.
- [24] Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Newbury Park, CA: Sage.
- [25] Makamure, C. & Jojo, Z. M. (2022). An analysis of errors for pre-service teachers in first order ordinary differential equations. *EURASIA Journal of Mathematics, Science and Technology Education*, 18(6), 1-11.
- [26] Morales, Z. A. (2014). Analysis of Students' Misconceptions and Error Patterns in Mathematics: *The Case of Fractions*.
- [27] National Council for Curriculum and Assessment & Ministry of Education (2019). *Common core Curriculum for Basic Schools*. Accra, Ghana.
- [28] Ndalichako, J. L. (2013). Analysis of pupils' difficulties in solving questions related to fractions: The case of primary school leaving examination in Tanzania. *Creative Education*, 4(9), 69-73.
- [29] Ojose, B. (2015). "Students' Misconceptions in Mathematics: Analysis of Remedies and What Research Says". *Ohio Journal of School Mathematics*, (72).
- [30] Okur, M., & Çakmak Gürel, Z. (2016). 6th and 7th grade secondary school students' misconceptions about fractions. *Erzincan University Education Faculty Journal*, 18(2), 922-952.
- [31] Önal, H., & Yorulmaz, A. (2017). The errors made by primary school fourth graders on fractions. *Journal of Research in Education and Society*, 4(1), 98-113.
- [32] Reinholda, F., Hoch, S., Werner, B., Richter-Gebert, J., & Reissa, K. (2020). Learning fractions with and without educational technology: What matters for high-achieving and low-achieving students? *Learning and Instruction*, 65, 10126.
- [33] Rosli, R., Goldsby, D., Onwuegbuzie, A. J., Capraro, M. M., Capraro, R. M., & Gonzalez, E. G. Y. (2020). Elementary Preservice Teachers' Knowledge, Perceptions and Attitudes towards Fractions: A Mixed-Analysis. *Journal on Mathematics Education*, 11(1), 59-76.
- [34] Ubah, I. J., & Bansilal, S. (2018). Pre-service primary Mathematics teachers' understanding of fractions: An action–process– object–schema perspective. *South African Journal of Childhood Education*, 8(2), 1-12.
- [35] Van de Walle, J. A., Karp, K., & Bay-Williams, J. M. (2019). *Elementary and middle school mathematics: Teaching developmentally* (10nd ed.). Boston: Pearson.
- [36] Van Hoof, J., Engelen, A. S., & Van Dooren, W. (2021). How robust are learners' misconceptions of fraction magnitude? An intervention study comparing the use of refutation and expository text. *Educational Psychology*, 41(5),524-543.
- [37] Wijaya, A. (2017). "The Relationships between Indonesian Fourth Graders' Difficulties in Fractions